Numeric Response Questions

Maxima and Minima

Q.1 The tangent to the curve $y = x^3 - 6x^2 + 9x + 4, 0 \leq x \leq 5$ has maximum slope at $x = k$ then find value of k.

Q.2 If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find a.

Q.3 Find the smallest value of the polynomial $x^3 - 18x^2 + 96x$ in the interval [0,9].

Q.4 The minimum value of $ax + by$, where $xy = r^2, (r, ab > 0)$ is $\lambda r\sqrt{ab}$ then find λ.

Q.5 Find the minimum value of the function $f(x) = 2|x - 2| + 5|x - 3|$ for all $x \in R$.

Q.6 If $y = a\log |x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$ then find value of a $+b$.

Q.7 If $ab = 2a + 3b, a > 0, b > 0$, then find the minimum value of ab,

Q.8 Using wire of length 20 m, boundary of a garden which is in the shape of a circular sector is to be made. If Area of the qarden is maximum, then find the radius of the sector.

Q.9 If maximum value of $\frac{1}{x^2-3x+6}$ is $\frac{a}{b}$ then find value of $a + b$.

Q.10 Difference between the greatest and the least values of the function $f(x) = x(inx - 2)$ on $[1, e^2]$ is λe^4 then find $\lambda + k$.

ANSWER KEY

1. 5.00	**2.** 2.00	**3.** 0.00	**4.** 2.00	**5.** 2.00	**6.** 1.50	**7.** 24.00
8. 5.00	**9.** 19.00	**10.** 2.00				

Hints & Solutions

1. $\frac{dy}{dx} = 3x^2 - 12x + 9$

Let $u = 3x^2 - 12x + 9$

$\frac{du}{dx} = 6x - 12 = 0 \Rightarrow x = 2$

at $x = 0$ $\quad u = 9$

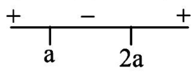
$x = 2$ $\quad u = -3$

$x = 5$ $\quad u = 24$

So $\frac{dy}{dx}$ is maximum at $x = 5$.

2. $f'(x) = 6x^2 - 18ax + 12a^2$

$= 6(x - a)(x - 2a)$



So $x = a$ is point of maxima and $x = 2a$ is point of minima

So $p = a$ and $q = 2a$

Now $p^2 = q \Rightarrow a^2 = 2a$

$\Rightarrow a = 2$

3. Let $f(x) = x^3 - 18x^2 + 96x$

Differentiating it w.r.t. to x

$f'(x) = 3x^2 - 36x + 96$

$= 3(x^2 - 12x + 32)$

$= 3(x - 4)(x - 8)$

Cleary at x = 8 f(x) is minimum.

$\min^m f(x) = 8(64 - 144 + 96) = 128$

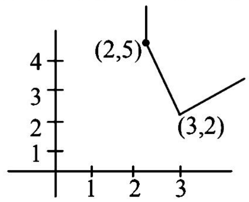
At $x = 0$, $f(x) = 0$

4. $AM \geq GM$

$\frac{ax + by}{2} \geq \sqrt{abxy}$

$ax + by \geq 2r\sqrt{ab}$

5. $f(x) = 2|x - 2| + 5|x - 3|$



minimum value = 2

6. $\frac{dy}{dx} = \frac{a}{x} + 2bx + 1$ or $\frac{dy}{dx} = 0$ at $x = -1, 2$

$\frac{a}{-1} + 2b(-1) + 1 = 0$ or $-a - 2b + 1 = 0$

$\frac{a}{2} + 4b + 1 = 0$ or $a + 8b + 2 = 0$

solving we get ; $a = 2, b = -\frac{1}{2}$

7. $ab = 2a + 3b \Rightarrow b = \frac{2a}{a-3}$

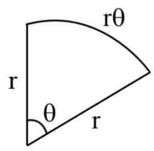
Now, $z = ab = \frac{2a^2}{a-3}$

So, $\frac{dz}{da} = \frac{2(a^2 - 6a)}{(a-3)^2} = 0 \Rightarrow a = 0, 6$

At $a = 6$, $\frac{d^2z}{da^2} = +ve$ $\quad$ So, $a = 6$, $b = 4$

$\therefore (ab)_{min} = 6 \times 4 = 24$

8.

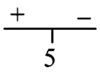


$2r + r\theta = 20 \Rightarrow \theta = \frac{20 - 2r}{r}$

Now area $A = \frac{r^2\theta}{2} = \frac{r^2(20-2r)}{2r}$

$A = 10r - r^2$

$\frac{dA}{dr} = 10 - 2r = 0 \Rightarrow r = 5$



So r = 5 is point of maxima

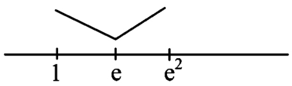
9. If will be maximum when $x^2 - 3x + 6$ be minimum

$\min(x^2 - 3x + 6) = -\frac{D}{4a} = -\frac{9-24}{4} = \frac{15}{4}$

10. $y = x(\ln x - 2)$

$y' = x\left(\frac{1}{x}\right) + (\ell n\, x - 2) = \ell n\, x - 1$

$\frac{dy}{dx} = \ell n\, x - 1 = 0 \Rightarrow x = e$



now $f(1) = -2$

$f(e) = -e$ (least)

$f(e^2) = 0$ (greatest)

$\therefore$ difference $= 0 - (-e) = e$